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# Coal Mill Foundation – A Finite Element Approach for Study of Dynamic Analysis

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**Abstract:** In general coal mill foundation consist of a block foundation with heavy concrete mass. Its dynamic behavior is studied using Barkan's[1] method by most of the engineering practitioners. Foundation is generally sized such that the foundation mass is about more than three times the mass of the equipment using the empirical guidelines. This method has limitations in providing wholistic details of foundation behavior like transient stage deformations and response for the exited forces, foundation response at specific areas/nodes of the foundation etc. Also, this method has got limitation in mode shapes. This method can provide only fundamental mode shapes. That is in three linear directions and three rotational directions. But in practical, Mode shapes exist with coupled directions with appropriate mass participations over and above the fundamental directions. In this study a three-dimensional Finite Element Model is used to study coal mill foundation dynamic response and it's behavior. This approach plugs all the limitations of Barkan's method. ANSYS software is used for modeling the foundation with three-dimensional Finite Elements. Appropriate soil structure interaction is resembled with suitable elements available in ANSYS element library. Dynamic forces from coal mill, gear box and motor which act at different frequencies and at different planes are applied on the model at appropriate nodes. Natural frequencies of the foundation system and displacement amplitudes from the forced vibration analysis are obtained using the above model. Mode shapes of the various modes are plotted. Foundation response for equipment startup and shut down conditions (transient stages) are studied by plotting displacement amplitudes w.r.t frequency variation.

**Keywords:** Coal Mill Foundation, Free Vibration Analysis, Block Foundation, Forced Vibration Analysis, ANSYS, Modes

## 1. Introduction

The foundation acts as a rigid body assumed normally in classical empirical methods for dynamic analysis, (e.g., Barkan 1962). However, the structure of a mill foundation and piers with large dimensions is flexible rather than a rigid body. Two stage approach can be used to carry out the dynamic analysis for mill foundations, namely the free vibration analysis and the forced vibration analysis. For a small ball mill, with a mill diameter less than 3.6 m and small dynamic loads, the method of free vibration analysis (also called modal analysis) can be used. The natural frequencies of foundation and piers can be calculated using the free vibration analysis to avoid the resonance. The natural frequency should be less than 0.7 $f_n$ or larger than 1.4 $f_n$, where $f_n$ is the operation frequency of the machine.

For large mill, the method based on forced vibration analysis along with free vibration analysis is recommended. The vibration amplitudes should be calculated to meet the requirement of allowable vibration limit. Dynamics analysis is difficult for the flexible mill foundations using standard analytical or numerical methods.

Classical empirical methods assume that the foundation acts as a rigid body. However, the structure of mill foundation and piers with large dimension is flexible rather than a rigid body. Numerical methods such as the general finite element method are also difficult to apply, as the direct simulation of damping is not possible. Damping is the dominant energy dissipation mechanism in most dynamically loaded foundation systems. The sub-structure method is used for dynamic analysis of mill foundation, that is, the structure and soil are considered separately.

The structure part (mat foundation and piers) are modelled by FEM model. The impedance of soil (stiffness and damping) are generated by the computer program, and then input to the FEM model as the base boundary condition. So the reasonable values of damping can be used with the help of program.

Evolution of software's based on Finite element method's such as Staad. Pro, SAP, ANSYS, Nastran etc.., paved the way to understand the behavior of these block foundations under dynamic loads under different operating conditions. From output results of these software's one can read the displacements / velocity amplitudes at different points on the foundation. This is helping practitioners to predict the behavior of foundation more precisely during equipment operating conditions.

The effects of soil-pile-structure interaction on dynamic response were discussed (e.g., Han) [1, 11]. Isolation systems can be useful to reduce foundation vibration [2-4]. And the resonant frequencies of the rotor and support system may cause very high amplitude vibration [5]. Despite of whole efforts were done to reduce damages of these types of structures, their lifetime is less than that of expected. Due to this fact, some studies were done to investigate this important parameter, assessing the life cycle of photovoltaic and wind power technologies [6]. Another study was done by Seo et al. [7] in which redundant systems consisting of units that alternate between operating and standby states periodically were analyzed. Moreover, some other studies were carried out to evaluate lifetime of concrete structures such as dams [8] and bridges [9, 10]. Studies on frequency and vibrations of medium range speed (900 rpm to 1500 rpm) equipment's foundations such as primary air fan and induced draft fan were carried out by Nulu Reddeppa et al. [12, 13]

# 2. Geometry and Basic Data

The geometry is considered as per machine manufacturer's foundation input drawings. It can be seen from the geometry that the mill foundation is a block foundation.

### 2.1. Material Properties

Material properties of M25 and M35 Grade concrete [14, 15] were given in table 1.

***Table 1.*** *Material properties of concrete.*

| Property | Value |
|---|---|
| Density, $kN/m^3$ | 25 |
| Characteristic Strength, $N/mm^2$ for M25 | 25 |
| Modulus of Elasticity, $N/mm^2$ for M25 | 34000 (Dynamic) |
| Characteristic Strength, $N/mm^2$ for M35 | 35 |
| Modulus of Elasticity, $N/mm^2$ for M35 | 40000 (Dynamic) |

### 2.2. MACHINE Data

The machine and foundation arrangement with the loads are specified in Machine drawings/documents. The salient features of the machine are summarized as below.

Total Mill Weight=1537.58 KN
Motor weight=92.50 KN
Self-weight of the Foundation ≅ 6194.51 KN
Mass ratio of Foundation to Machine=3.80 > 2.5

### 2.3. Software Reference

ANSYS/Structural 13.0 ver, ANSYS, Inc., USA software package is renowned and is in practice in the industry both nationally and internationally for similar type of structural dynamic analysis of foundations. Consistent units of Newton, Meter are used in the present analysis.

# 3. FEM Modeling of Structure

Solid brick finite elements are used to represent the geometry of mill foundation for dynamic analysis. The solid model is built in ANSYS software based on this geometry and then the finite element is created by meshing using solid elements "solid186" of ANSYS element library. The volume mesh contains brick element shapes of the solid186 shown below.

Solid186 element has three degrees of freedom in three linear directions. The foundation is provided with soil spring supports estimated as per the soil characteristics of the site.

The following sections describe the analysis/design methodology including loading consideration and finally summarize the results.

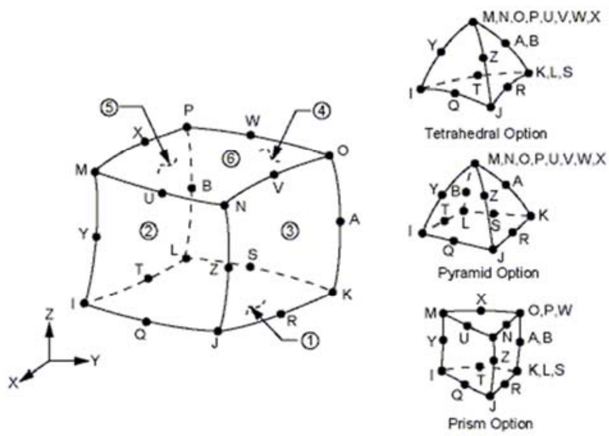


***Figure 1.*** *Solid186 Element Geometry.*

# 4. Modal Analysis – Natural Frequencies

The Mode-Frequency analysis for natural frequency and mode shape determination is carried out in ANSYS. The assumptions made in this analysis are the structure has no time varying forces, displacements, pressures, or temperatures applied, which means that this is free vibration analysis.

There is no damping in the structural system.

The structure has constant stiffness and mass effects.

3D MASS 21 element (from ANSYS element library) is used to represent machine mass application points on foundation block. COMBIN14 is used to model lateral soil springs on the foundation.

The natural frequencies are requested for first 25 modes of vibration. But only 11 modes are present in the foundation system and they are tabulated below. Refer to ANSYS output in Section 5 for natural frequency output and mode participation factor table.

Dynamic stiffness Calculations:

Vertical Stiffness is governed by the soil at the bottom of the pile which is dense sand.

Grade of concrete=30 MPa

Young's Modulus Est=241.57 MPa

Length of the pile L=49 m

Diameter of the pile D=0.75 m

Radius of the pile R=0.375 m

Pile Slenderness=L/R=130.7

Weighted average Young's modulus of the soil along the pile length to be calculated as per site condition. However, the same is considered as 241.57 MPa in the present analysis.

$$\text{Ep}=5000 * \sqrt{fck}$$

$$\text{Ep}=27386 \text{ MPa}$$

Poisson's Ratio μ=0.281

Unit weight of the concrete=25 KN/m$^3$

Shear Modulus $Gs = \frac{Es}{2(1+\mu)}$ =94.29 Mpa

Unit weight of the soil, γ s=18 KN/m$^3$

Shear wave velocity $Vs = \sqrt{\frac{Gs\ g}{\Upsilon s}}$ =7.17 m/sec

Compression wave velocity

$$Vc = \sqrt{\frac{Ep\ g}{\Upsilon p}} = 103.66 \text{ m/sec}$$

$$\frac{V_s}{V_c} = 0.069$$

Stiffness Factor f18,1=0.036 [11]

Radius of the pile=375 mm

Vertical Stiffness Kz=( $\frac{E_P.A}{\gamma_o}$) × $f18{,}1$ [11] =1161954283 N/m

Since the Lateral Stiffness is governed by the top of the pile which is dense Sand.

Young's Modulus Es=123.52 Mpa

Poisson's Ratio μ=0.258

Shear Modulus $G_S = \frac{E_S}{2(1+\mu)}$ =49.094 Mpa

Shear wave velocity, $Vs = \sqrt{\frac{Gs\ g}{\Upsilon s}}$ =5.17 m/sec

Compression wave velocity, $Vc = \sqrt{\frac{Ep\ g}{\Upsilon p}}$ =103.664 m/sec

$$\frac{V_s}{V_c} = 0.049$$

Hence from table 2 extracted from reference [11].

$$f_{11,1}=0.03591 \text{ [11]}$$

Moment of Inertia, l=15537806920 mm4

Horizontal Stiffness $K_X = (\frac{E_P.I}{\gamma^3.O}) . f_{11,1}$ [11]=289757370.5 N/m

## 5. FEM Model Sketches

Below sketches shows the FEM model of coal mill foundation system generated using the ANSYS software.

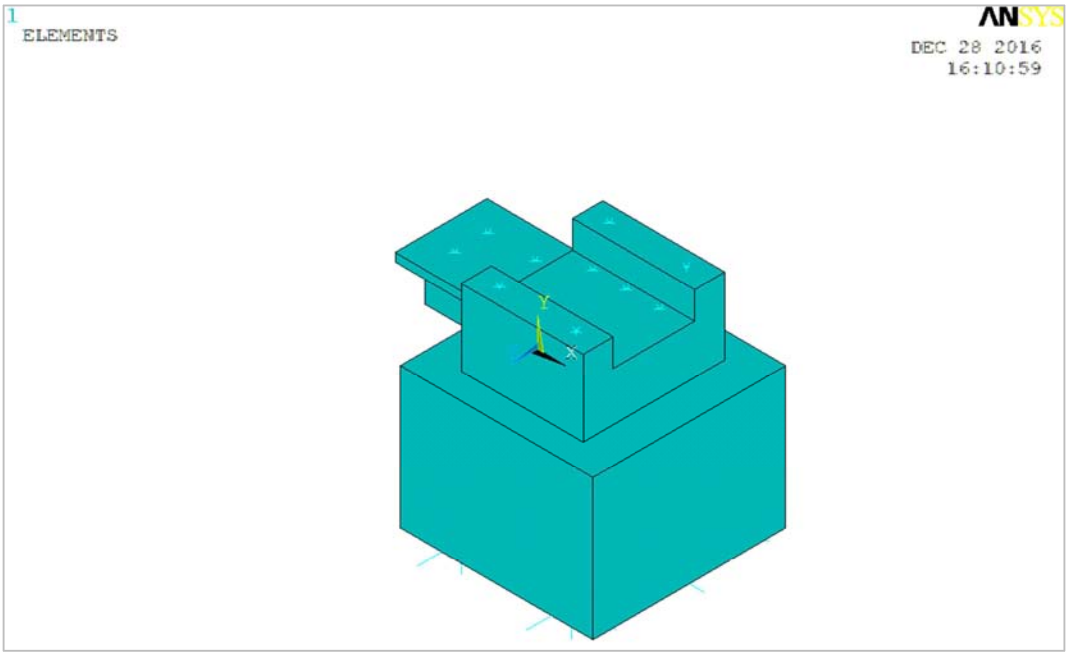


***Figure 2.*** *ANYS – Solid Model.*

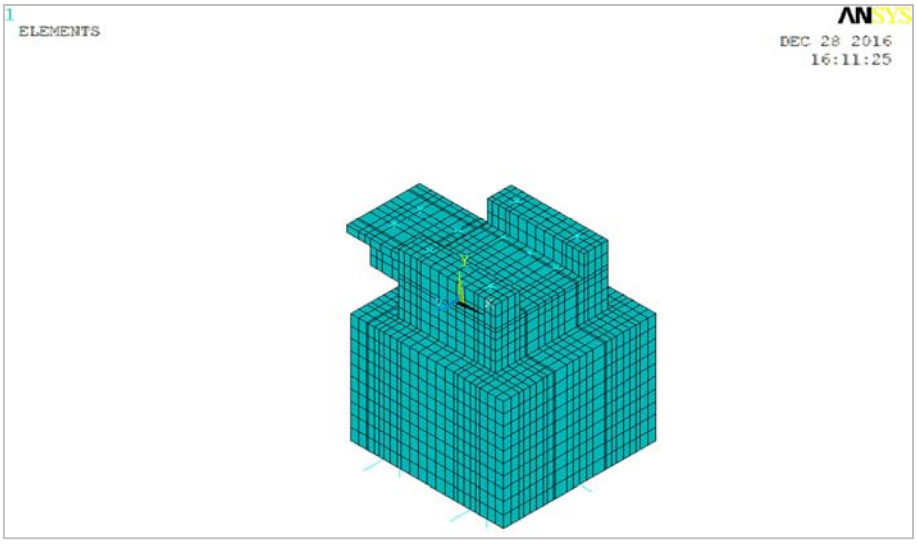


***Figure 3.*** *ANSYS – Solid Model – Element Mesh.*

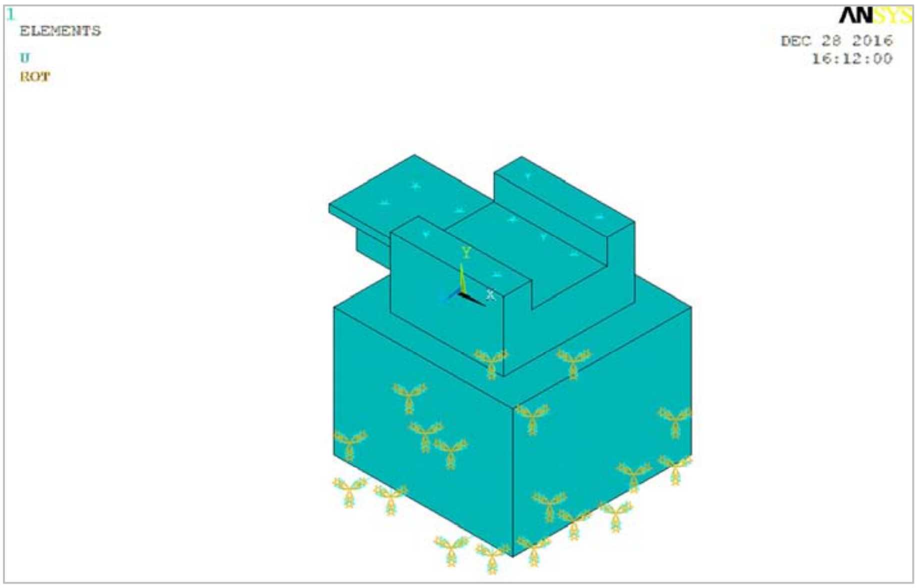


***Figure 4.*** *ANSYS-Solid Model-Applied Boundary Conditions.*

***Table 2.*** *Stiffness and Damping Parameters for Piles.*

*Values of f 11, 1; f 11, 2; f 7, 1; f 7,2; f 9, 1; f 9,2*
*For I/$\gamma_0$>25 (after ref. 10)**

| Ve | VB/ Vc | f 11, 1 | f 11, 2 | f 7,1 | f 7,2 | f 9,1 | f 9, 2 |
|---|---|---|---|---|---|---|---|
| Concrete Piles (ɣ B/ ɣ D=0.7) | | | | | | | |
| 0.4 | 0.01 | 0.0036 | 0.0084 | 0.202 | 0.139 | -0.0194 | -0.028 |
| | 0.03 | 0.0185 | 0.0438 | 0.349 | 0.243 | -0.0582 | 0.0848 |
| | 0.05 | 0.0397 | 0.0942 | 0.45 | 0.314 | -0.097 | -0.141 |
| 0.25 | 0.01 | 0.0032 | 0.0076 | 0.195 | 0.135 | -0.0181 | -0.0262 |
| | 0.03 | 0.0166 | 0.0395 | 0.337 | 0.235 | -0.0543 | -0.0793 |
| | 0.05 | 0.0358 | 0.085 | 0.435 | 0.304 | -0.905 | 0.1321 |
| Timber Piles (ɣ B/ ɣ D=2) | | | | | | | |
| 0.4 | 0.01 | 0.0082 | 0.0183 | 0.265 | 0.176 | -0.0336 | -0.0466 |
| | 0.03 | 0.0425 | 0.0949 | 0.459 | 0.305 | -0.101 | -0.14 |
| | 0.05 | 0.0914 | 0.204 | 0.592 | 0.394 | -0.168 | -0.233 |
| 0.25 | 0.01 | 0.0074 | 0.0165 | 0.256 | 0.169 | -0.0315 | -0.0434 |
| | 0.03 | 0.0385 | 0.0854 | 0.444 | 0.293 | 0.0945 | -0.1301 |
| | 0.05 | 0.0828 | 0.1838 | 0.573 | 0.379 | -0.1575 | -0.2168 |

* Values are appropriate for $\alpha_0$=0.3, but are approximately valid (±10%) for $0.1 \leq \alpha_0 \leq 0.8$. Reproduced by permission of the National Research Council of Canada from the Canadian Geotechnical Journal, Vol 11 (1974) p, 584.

# 6. Analysis Output

Referring to following tables the natural frequencies of the foundation system are identified in all linear and rotational modes for bot upper and lower young's modulus of concrete.

### *6.1. ANSYS – Natural Frequencies for M25 Grade of Concrete*

***Table 3.*** *Participation factor calculation X direction.*

| MODE | FREQUENCY | PERIOD | PARTICIPATION FACTOR | RATIO | EFFECTIVE MASS | CUMULATIVE MASS FRACTION | RATIO EFF. MASS TO TOTAL MASS |
|---|---|---|---|---|---|---|---|
| 1 | 4.50763 | 0.22185 | 810.35 | 1 | 656660 | 0.827263 | 0.827222 |
| 2 | 4.65688 | 0.21474 | -3.46E-03 | 0.000004 | 1.20E-05 | 0.827263 | 1.51E-11 |
| 3 | 8.24302 | 0.12131 | -8.41E-03 | 0.00001 | 7.07E-05 | 0.827263 | 8.90E-11 |
| 4 | 12.8231 | 7.80E-02 | -102.38 | 0.126335 | 10480.7 | 0.840466 | 1.32E-02 |
| 5 | 14.0964 | 7.09E-02 | 355.85 | 0.439139 | 126633 | 0.999998 | 0.159524 |
| 6 | 14.3241 | 6.98E-02 | -0.11356 | 0.00014 | 1.29E-02 | 0.999998 | 1.62E-08 |
| 7 | 40.3484 | 2.48E-02 | 4.31E-02 | 0.000053 | 1.86E-03 | 0.999998 | 2.34E-09 |
| 8 | 43.0244 | 2.32E-02 | 1.82E-02 | 0.000022 | 3.32E-04 | 0.999998 | 4.18E-10 |
| 9 | 43.0941 | 2.32E-02 | -1.0987 | 0.001356 | 1.20705 | 1 | 1.52E-06 |
| 10 | 48.4608 | 2.06E-02 | -1.21E-03 | 0.000001 | 1.46E-06 | 1 | 1.83E-12 |
| 11 | 49.7996 | 2.01E-02 | -2.84E-04 | 0 | 8.06E-08 | 1 | 1.02E-13 |
| sum | | | | | 793775 | | 0.999951 |

***Table 4.*** *Participation factor calculation Y direction.*

| MODE | FREQUENCY | PERIOD | PARTICIPATION FACTOR | RATIO | EFFECTIVE MASS | CUMULATIVE MASS FRACTION | RATIO EFF. MASS TO TOTAL MASS |
|---|---|---|---|---|---|---|---|
| 1 | 4.50763 | 0.22185 | 20.982 | 0.024192 | 440.236 | 5.55E-04 | 5.55E-04 |
| 2 | 4.65688 | 0.21474 | -3.27E-02 | 0.000038 | 1.07E-03 | 5.55E-04 | 1.34E-09 |
| 3 | 8.24302 | 0.12131 | 1.66E-03 | 0.000002 | 2.76E-06 | 5.55E-04 | 3.48E-12 |
| 4 | 12.8231 | 7.80E-02 | 867.31 | 1 | 752225 | 0.948705 | 0.947609 |
| 5 | 14.0964 | 7.09E-02 | 201.69 | 0.232545 | 40678.4 | 0.999978 | 5.12E-02 |
| 6 | 14.3241 | 6.98E-02 | -0.24746 | 0.000285 | 6.12E-02 | 0.999978 | 7.71E-08 |
| 7 | 40.3484 | 2.48E-02 | 4.1316 | 0.004764 | 17.0703 | 1 | 2.15E-05 |
| 8 | 43.0244 | 2.32E-02 | -5.48E-04 | 0.000001 | 3.00E-07 | 1 | 3.78E-13 |
| 9 | 43.0941 | 2.32E-02 | 3.20E-02 | 0.000037 | 1.03E-03 | 1 | 1.29E-09 |
| 10 | 48.4608 | 2.06E-02 | 3.92E-04 | 0 | 1.53E-07 | 1 | 1.93E-13 |
| 11 | 49.7996 | 2.01E-02 | -1.90E-02 | 0.000022 | 3.60E-04 | 1 | 4.53E-10 |
| sum | | | | | 793361 | | 0.99943 |

***Table 5.*** *Participation factor calculation Z direction.*

| MODE | FREQUENCY | PERIOD | PARTICIPATION FACTOR | RATIO | EFFECTIVE MASS | CUMULATIVE MASS FRACTION | RATIO EFF. MASS TO TOTAL MASS |
|---|---|---|---|---|---|---|---|
| 1 | 4.50763 | 0.22185 | 3.83E-03 | 0.000005 | 1.46E-05 | 1.84E-11 | 1.84E-11 |
| 2 | 4.65688 | 0.21474 | 817.37 | 1 | 668089 | 0.841682 | 0.84162 |
| 3 | 8.24302 | 0.12131 | -30.978 | 0.0379 | 959.642 | 0.842891 | 1.21E-03 |
| 4 | 12.8231 | 7.80E-02 | 9.91E-02 | 0.000121 | 9.83E-03 | 0.842891 | 1.24E-08 |
| 5 | 14.0964 | 7.09E-02 | 0.13972 | 0.000171 | 1.95E-02 | 0.842891 | 2.46E-08 |
| 6 | 14.3241 | 6.98E-02 | 353.12 | 0.432015 | 124690 | 0.99998 | 0.157078 |
| 7 | 40.3484 | 2.48E-02 | -3.96E-03 | 0.000005 | 1.57E-05 | 0.99998 | 1.98E-11 |
| 8 | 43.0244 | 2.32E-02 | 6.05E-02 | 0.000074 | 3.66E-03 | 0.99998 | 4.61E-09 |
| 9 | 43.0941 | 2.32E-02 | 8.05E-04 | 0.000001 | 6.49E-07 | 0.99998 | 8.17E-13 |
| 10 | 48.4608 | 2.06E-02 | -0.27049 | 0.000331 | 7.32E-02 | 0.99998 | 9.22E-08 |
| 11 | 49.7996 | 2.01E-02 | 3.9987 | 0.004892 | 15.9894 | 1 | 2.01E-05 |
| sum | | | | | 793755 | | 0.999926 |

***Table 6.*** *Participation factor calculation ROT X direction.*

| MODE | FREQUENCY | PERIOD | PARTICIPATION FACTOR | RATIO | EFFECTIVE MASS | CUMULATIVE MASS FRACTION |
|---|---|---|---|---|---|---|
| 1 | 4.50763 | 0.22185 | -68.175 | 0.020569 | 4647.88 | 2.19E-04 |
| 2 | 4.65688 | 0.21474 | 3314.4 | 1 | 1.10E+07 | 0.518316 |

| MODE | FREQUENCY | PERIOD | PARTICIPATION FACTOR | RATIO | EFFECTIVE MASS | CUMULATIVE MASS FRACTION |
|---|---|---|---|---|---|---|
| 3 | 8.24302 | 0.12131 | -80.174 | 0.02419 | 6427.87 | 0.518619 |
| 4 | 12.8231 | 7.80E-02 | -2819 | 0.850516 | 7.95E+06 | 0.893399 |
| 5 | 14.0964 | 7.09E-02 | -655.98 | 0.197919 | 430316 | 0.913693 |
| 6 | 14.3241 | 6.98E-02 | -1351.9 | 0.407873 | 1.83E+06 | 0.999884 |
| 7 | 40.3484 | 2.48E-02 | -13.408 | 0.004045 | 179.767 | 0.999893 |
| 8 | 43.0244 | 2.32E-02 | -5.33E-02 | 0.000016 | 2.84E-03 | 0.999893 |
| 9 | 43.0941 | 2.32E-02 | -0.10497 | 0.000032 | 1.10E-02 | 0.999893 |
| 10 | 48.4608 | 2.06E-02 | -1.56E-03 | 0 | 2.43E-06 | 0.999893 |
| 11 | 49.7996 | 2.01E-02 | -47.679 | 0.014385 | 2273.25 | 1 |
| sum | | | | | 2.12E+07 | |

***Table 7.*** *Participation factor calculation ROT Y direction.*

| MODE | FREQUENCY | PERIOD | PARTICIPATION FACTOR | RATIO | EFFECTIVE MASS | CUMULATIVE MASS FRACTION |
|---|---|---|---|---|---|---|
| 1 | 4.50763 | 0.22185 | 2633.7 | 0.662762 | 6.94E+06 | 0.209739 |
| 2 | 4.65688 | 0.21474 | -3973.8 | 1 | 1.58E+07 | 0.687228 |
| 3 | 8.24302 | 0.12131 | 2378 | 0.598404 | 5.65E+06 | 0.858211 |
| 4 | 12.8231 | 7.80E-02 | -333.22 | 0.083853 | 111034 | 0.861569 |
| 5 | 14.0964 | 7.09E-02 | 1155.8 | 0.290849 | 1.34E+06 | 0.901961 |
| 6 | 14.3241 | 6.98E-02 | -1800.3 | 0.453046 | 3.24E+06 | 0.999966 |
| 7 | 40.3484 | 2.48E-02 | 0.15947 | 0.00004 | 2.54E-02 | 0.999966 |
| 8 | 43.0244 | 2.32E-02 | -8.6294 | 0.002172 | 74.4664 | 0.999968 |
| 9 | 43.0941 | 2.32E-02 | -3.6962 | 0.00093 | 13.662 | 0.999969 |
| 10 | 48.4608 | 2.06E-02 | 27.759 | 0.006986 | 770.573 | 0.999992 |
| 11 | 49.7996 | 2.01E-02 | -16.239 | 0.004086 | 263.697 | 1 |
| sum | | | | | 3.31E+07 | |

***Table 8.*** *Participation factor calculation ROT Z direction.*

| MODE | FREQUENCY | PERIOD | PARTICIPATION FACTOR | RATIO | EFFECTIVE MASS | CUMULATIVE MASS FRACTION |
|---|---|---|---|---|---|---|
| 1 | 4.50763 | 0.22185 | -3243.1 | 0.789793 | 1.05E+07 | 0.322411 |
| 2 | 4.65688 | 0.21474 | -0.14841 | 0.000036 | 2.20E-02 | 0.322411 |
| 3 | 8.24302 | 0.12131 | 6.88E-02 | 0.000017 | 4.73E-03 | 0.322411 |
| 4 | 12.8231 | 7.80E-02 | 4106.3 | 1 | 1.69E+07 | 0.839283 |
| 5 | 14.0964 | 7.09E-02 | 2289.7 | 0.557598 | 5.24E+06 | 0.999986 |
| 6 | 14.3241 | 6.98E-02 | -1.6725 | 0.000407 | 2.79734 | 0.999986 |
| 7 | 40.3484 | 2.48E-02 | 20.82 | 0.00507 | 433.463 | 0.999999 |
| 8 | 43.0244 | 2.32E-02 | 8.56E-02 | 0.000021 | 7.33E-03 | 0.999999 |
| 9 | 43.0941 | 2.32E-02 | -5.4951 | 0.001338 | 30.196 | 1 |
| 10 | 48.4608 | 2.06E-02 | -4.09E-05 | 0 | 1.67E-09 | 1 |
| 11 | 49.7996 | 2.01E-02 | -9.71E-02 | 0.000024 | 9.43E-03 | 1 |
| sum | | | | | 3.26E+07 | |

The natural frequency of the foundation system in each direction is corresponding to highest mass participation mode and the same are mentioned below:

The natural frequency along the rotor direction (X-dir)=4.50763 Hz: 1$^{st}$ Mode

The natural frequency across the rotor direction (Z-dir)=4.65688 Hz: 2$^{nd}$ Mode

The natural frequency in vertical direction (Y-dir)=12.8231 Hz: 4$^{th}$ Mode

The operating frequency of the motor=16.47 Hz

The operating frequency of the mill=0.55 Hz

The natural frequencies are far away from the Mill frequency range: 0.44Hz (0.8*0.55) To 0.66Hz (1.2*0.55) and Motor frequency range: 13.176Hz (0.8*16.47) To 19.764Hz (1.2*16.47) as per IS: 2974 Part-4. [15]

Also, it may be noted that, the motor rotor weight is 17KN which is only 364$^{th}$ part of the foundation weight (6194.51KN). In general, if the rotating part weight is one hundredth part or less of its supporting foundation weight, then its dynamic effect on foundation are negligible. Hence motor dynamic effects can be ignored. Foundation frequencies are studied for higher grade concrete (M35) to verify a better frequency separation in the following section.

### *6.2. ANSYS – Natural Frequencies for M35 Grade of Concrete*

***Table 9.*** *Participation factor calculation X direction.*

| MODE | FREQUENCY | PERIOD | PARTICIPATION FACTOR | RATIO | EFFECTIVE MASS | CUMULATIVE MASS FRACTION | RATIO EFF. MASS TO TOTAL MASS |
|---|---|---|---|---|---|---|---|
| 1 | 4.57502 | 0.21858 | 812 | 1 | 659351 | 0.830641 | 0.830612 |
| 2 | 4.72611 | 0.21159 | -5.00E-03 | 0.000006 | 2.50E-05 | 0.830641 | 3.15E-11 |
| 3 | 8.30028 | 0.12048 | -9.49E-03 | 0.000012 | 9.00E-05 | 0.830641 | 1.13E-10 |
| 4 | 13.0673 | 7.65E-02 | -109.03 | 0.134275 | 11888 | 0.845618 | 1.50E-02 |
| 5 | 14.2629 | 7.01E-02 | 350.07 | 0.431112 | 122546 | 0.999999 | 0.154376 |
| 6 | 14.4963 | 6.90E-02 | -0.1121 | 0.000138 | 1.26E-02 | 0.999999 | 1.58E-08 |
| 7 | 43.7634 | 2.29E-02 | 3.62E-02 | 0.000045 | 1.31E-03 | 0.999999 | 1.65E-09 |
| 8 | 46.6651 | 2.14E-02 | 1.53E-02 | 0.000019 | 2.33E-04 | 0.999999 | 2.94E-10 |
| 9 | 46.7418 | 2.14E-02 | -0.93113 | 0.001147 | 0.867009 | 1 | 1.09E-06 |
| sum | | | | | 7.94E+05 | | 1.00E+00 |

***Table 10.*** *Participation factor calculation Y direction.*

| MODE | FREQUENCY | PERIOD | PARTICIPATION FACTOR | RATIO | EFFECTIVE MASS | CUMULATIVE MASS FRACTION | RATIO EFF. MASS TO TOTAL MASS |
|---|---|---|---|---|---|---|---|
| 1 | 4.57502 | 0.21858 | 20.673 | 0.023962 | 427.359 | 5.39E-04 | 5.38E-04 |
| 2 | 4.72611 | 0.21159 | -2.96E-02 | 0.000034 | 8.78E-04 | 5.39E-04 | 1.11E-09 |
| 3 | 8.30028 | 0.12048 | 1.46E-03 | 0.000002 | 2.14E-06 | 5.39E-04 | 2.70E-12 |
| 4 | 13.0673 | 7.65E-02 | 862.73 | 1 | 744306 | 0.938585 | 9.38E-01 |
| 5 | 14.2629 | 7.01E-02 | 220.72 | 0.255839 | 48717.6 | 0.999984 | 6.14E-02 |
| 6 | 14.4963 | 6.90E-02 | -0.25629 | 0.000297 | 6.57E-02 | 0.999984 | 8.27E-08 |
| 7 | 43.7634 | 2.29E-02 | 3.61E+00 | 0.004181 | 1.30E+01 | 1 | 1.64E-05 |
| 8 | 46.6651 | 2.14E-02 | -5.04E-04 | 0.000001 | 2.54E-07 | 1 | 3.20E-13 |
| 9 | 46.7418 | 2.14E-02 | 3.00E-02 | 0.000035 | 8.98E-04 | 1 | 1.13E-09 |
| sum | | | | | 793464 | | 0.99956 |

***Table 11.*** *Participation factor calculation Z direction.*

| MODE | FREQUENCY | PERIOD | PARTICIPATION FACTOR | RATIO | EFFECTIVE MASS | CUMULATIVE MASS FRACTION | RATIO EFF. MASS TO TOTAL MASS |
|---|---|---|---|---|---|---|---|
| 1 | 4.57502 | 0.21858 | 5.26E-03 | 0.000006 | 2.76E-05 | 3.48E-11 | 3.48E-11 |
| 2 | 4.72611 | 0.21159 | 8.19E+02 | 1 | 6.71E+05 | 8.45E-01 | 8.45E-01 |
| 3 | 8.30028 | 0.12048 | -3.18E+01 | 0.038776 | 1.01E+03 | 8.46E-01 | 1.27E-03 |
| 4 | 13.0673 | 7.65E-02 | 9.60E-02 | 0.000117 | 9.21E-03 | 0.846343 | 1.16E-08 |
| 5 | 14.2629 | 7.01E-02 | 0.14038 | 0.000171 | 1.97E-02 | 0.846343 | 2.48E-08 |
| 6 | 14.4963 | 6.90E-02 | 349.24 | 0.426412 | 1.22E+05 | 1 | 1.54E-01 |
| 7 | 43.7634 | 2.29E-02 | -3.36E-03 | 0.000004 | 1.13E-05 | 1 | 1.42E-11 |
| 8 | 46.6651 | 2.14E-02 | 5.06E-02 | 0.000062 | 2.56E-03 | 1 | 3.22E-09 |
| 9 | 46.7418 | 2.14E-02 | 6.63E-04 | 0.000001 | 4.40E-07 | 1 | 5.54E-13 |
| sum | | | | | 793760 | | 0.999932 |

***Table 12.*** *Participation factor calculation ROT X direction.*

| MODE | FREQUENCY | PERIOD | PARTICIPATION FACTOR | RATIO | EFFECTIVE MASS | CUMULATIVE MASS FRACTION |
|---|---|---|---|---|---|---|
| 1 | 4.57502 | 0.21858 | -67.165 | 0.020304 | 4511.12 | 2.13E-04 |
| 2 | 4.72611 | 0.21159 | 3307.9 | 1 | 1.09E+07 | 0.516283 |
| 3 | 8.30028 | 0.12048 | -81.298 | 0.024577 | 6609.29 | 0.516594 |
| 4 | 13.0673 | 7.65E-02 | -2804.1 | 0.847694 | 7.86E+06 | 0.887434 |
| 5 | 14.2629 | 7.01E-02 | -717.85 | 0.217011 | 515314 | 0.911738 |
| 6 | 14.4963 | 6.90E-02 | -1368 | 0.41354 | 1.87E+06 | 0.999994 |
| 7 | 43.7634 | 2.29E-02 | -11.706 | 0.003539 | 137.036 | 1 |
| 8 | 46.6651 | 2.14E-02 | -3.85E-02 | 0.000012 | 1.48E-03 | 1 |
| 9 | 46.7418 | 2.14E-02 | -9.79E-02 | 0.00003 | 9.59E-03 | 1 |
| sum | | | | | 2.12E+07 | |

***Table 13.*** *Participation factor calculation ROT Y direction.*

| MODE | FREQUENCY | PERIOD | PARTICIPATION FACTOR | RATIO | EFFECTIVE MASS | CUMULATIVE MASS FRACTION |
|---|---|---|---|---|---|---|
| 1 | 4.57502 | 0.21858 | 2639.1 | 0.662952 | 6.96E+06 | 0.210599 |
| 2 | 4.72611 | 0.21159 | -3980.8 | 1 | 1.58E+07 | 0.689772 |
| 3 | 8.30028 | 0.12048 | 2381.9 | 0.598334 | 5.67E+06 | 0.861317 |

| MODE | FREQUENCY | PERIOD | PARTICIPATION FACTOR | RATIO | EFFECTIVE MASS | CUMULATIVE MASS FRACTION |
|---|---|---|---|---|---|---|
| 4 | 13.0673 | 7.65E-02 | -354.84 | 0.089136 | 125908 | 0.865124 |
| 5 | 14.2629 | 7.01E-02 | 1137 | 0.285613 | 1.29E+06 | 0.904213 |
| 6 | 14.4963 | 6.90E-02 | -1779.8 | 0.447099 | 3.17E+06 | 0.999998 |
| 7 | 43.7634 | 2.29E-02 | 0.13412 | 0.000034 | 1.80E-02 | 0.999998 |
| 8 | 46.6651 | 2.14E-02 | -7.411 | 0.001862 | 54.9231 | 1 |
| 9 | 46.7418 | 2.14E-02 | -3.1327 | 0.000787 | 9.8141 | 1 |
| sum | | | | | 3.31E+07 | |

***Table 14.*** *Participation factor calculation ROT Z direction.*

| MODE | FREQUENCY | PERIOD | PARTICIPATION FACTOR | RATIO | EFFECTIVE MASS | CUMULATIVE MASS FRACTION |
|---|---|---|---|---|---|---|
| 1 | 4.57502 | 0.21858 | -3238.8 | 0.79965 | 1.05E+07 | 0.321525 |
| 2 | 4.72611 | 0.21159 | -0.12721 | 0.000031 | 1.62E-02 | 0.321525 |
| 3 | 8.30028 | 0.12048 | 6.48E-02 | 0.000016 | 4.20E-03 | 0.321525 |
| 4 | 13.0673 | 7.65E-02 | 4050.3 | 1 | 1.64E+07 | 0.824347 |
| 5 | 14.2629 | 7.01E-02 | 2393.8 | 0.591027 | 5.73E+06 | 0.999989 |
| 6 | 14.4963 | 6.90E-02 | -1.7212 | 0.000425 | 2.96264 | 0.999989 |
| 7 | 43.7634 | 2.29E-02 | 18.168 | 0.004485 | 330.061 | 0.999999 |
| 8 | 46.6651 | 2.14E-02 | 7.35E-02 | 0.000018 | 5.40E-03 | 0.999999 |
| 9 | 46.7418 | 2.14E-02 | -4.7747 | 0.001179 | 22.7974 | 1 |
| sum | | | | | 3.26E+07 | |

The natural frequency along the rotor direction (X-dir) =4.57502 Hz: 1$^{st}$ Mode

The natural frequency across the rotor direction (Z-dir) =4.72611 Hz: 2$^{nd}$ Mode

The natural frequency in vertical direction (Y-dir) =13.0673 Hz: 4$^{th}$ Mode

The operating frequency of the motor=988/60 =16.47 Hz

The operating frequency of the mill=33/60 =0.55 Hz

The natural frequencies are far away from the Mill frequency range: 0.44Hz (0.8*0.55) To 0.66Hz (1.2*0.55) and Motor frequency range: 13.176Hz (0.8*16.47) To 19.764Hz (1.2*16.47) As per IS: 2974 Part-4. [15]

Also, it may be noted that, the motor rotor weight is 17kN which is only 364$^{th}$ part of the foundation weight (6194.51KN). In general, if the rotating part weight is one hundredth part or less of its supporting foundation weight, then its dynamic effect on foundation are negligible. Hence motor dynamic effects can be ignored.

M35 grade frequencies (i.e. 13.0673 Hz) are very close to 0.8 times of motor frequency (i.e. 0.8 x 16.47=13.176Hz). Therefore, M25 grade of concrete is used for foundation which is having better frequency separation.

### *6.3. ANSYS – Mode Shape Plots for M25 Grade*

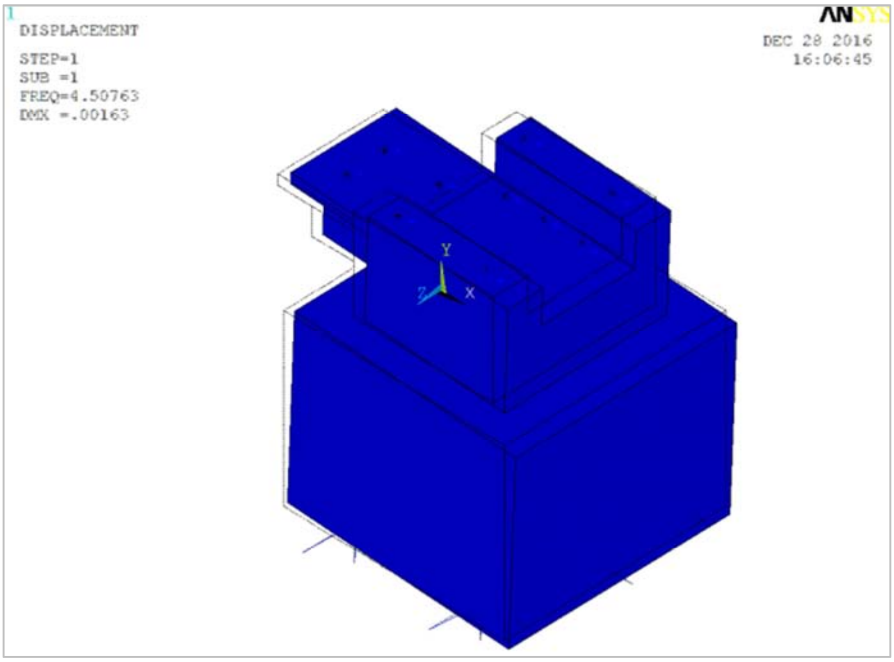


***Figure 5.*** *Mode No. 1: Deformation Plot.*

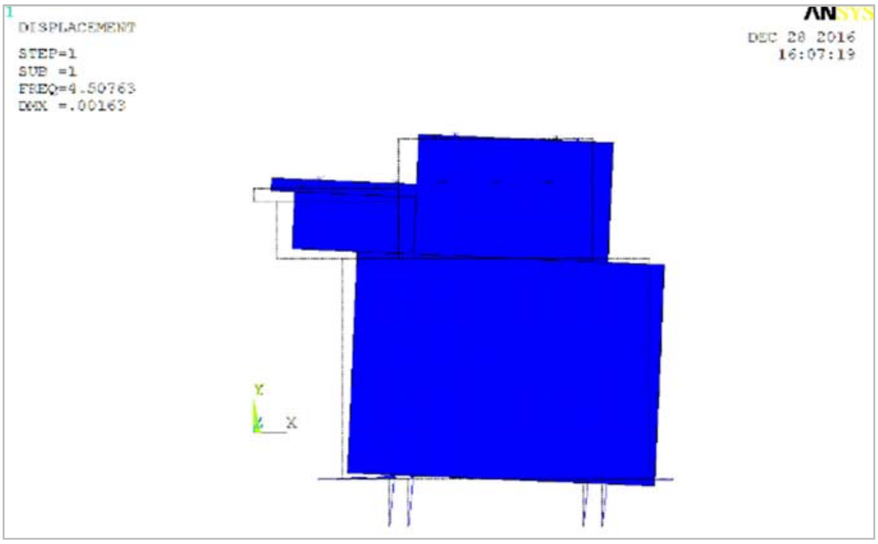


***Figure 6.*** *Mode No. 1: Deformation Plot – Side View.*

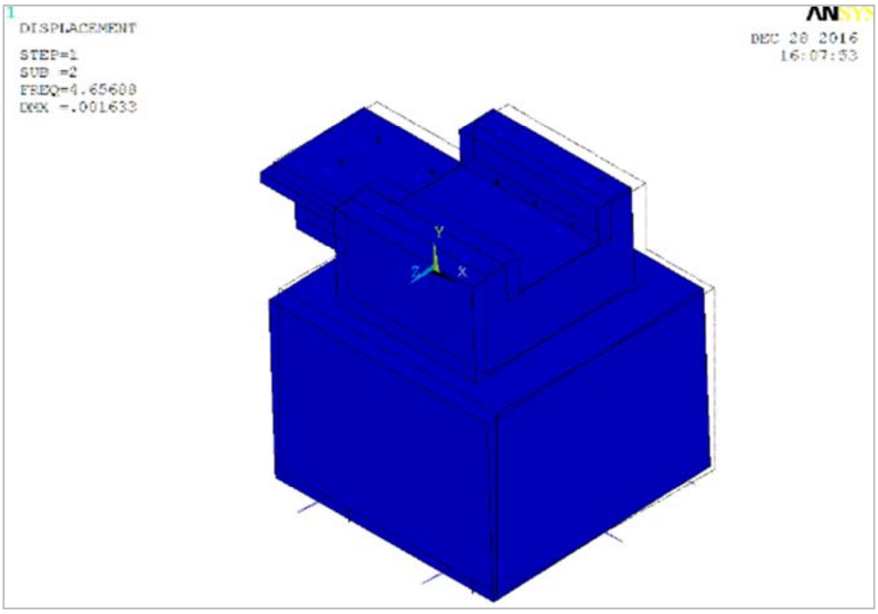


***Figure 7.*** *Mode No. 2: Deformation Plot.*

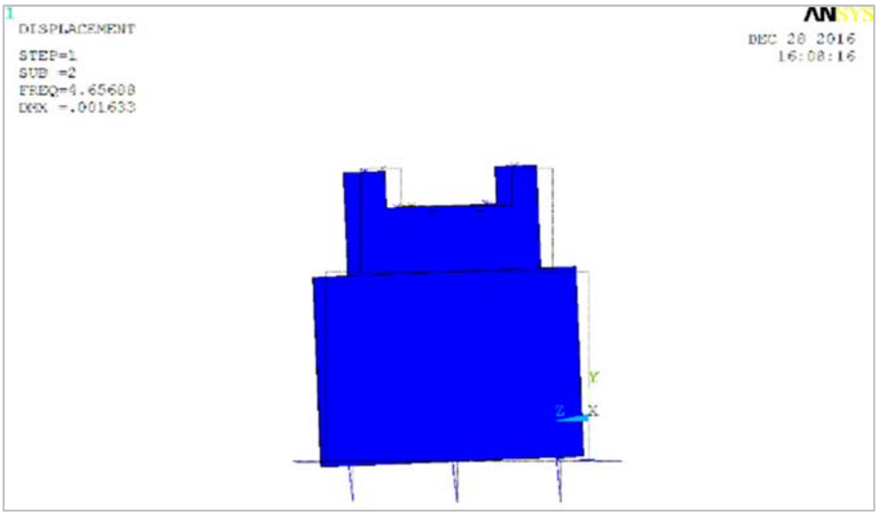


***Figure 8.*** *Mode No. 2: Deformation Plot - Side View.*

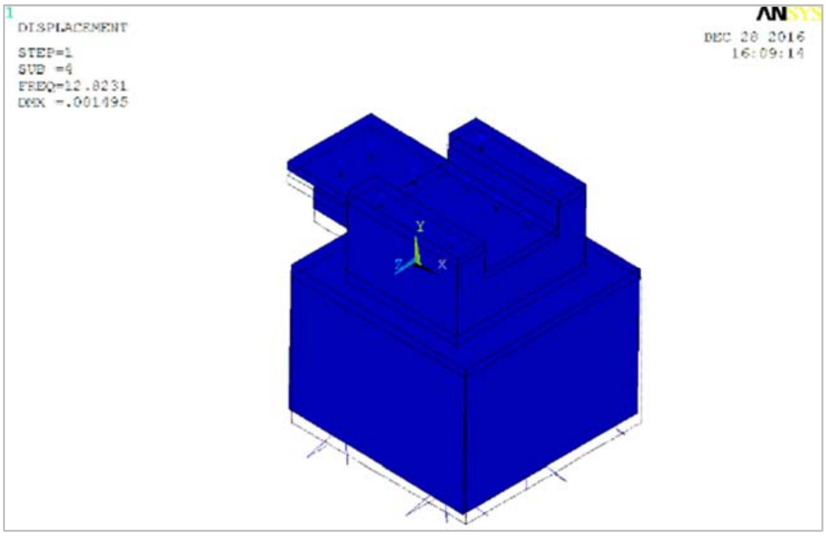


***Figure 9.*** *Mode No. 4 Deformation Plot.*

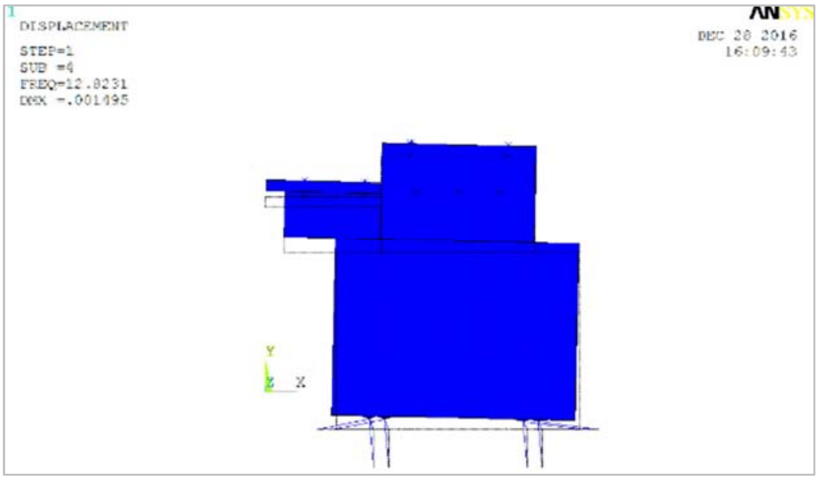


***Figure 10.*** *Mode No. 4 Deformation Plot – Side View.*

### *6.4. ANSYS – Amplitude Output*

*Dynamic Force Calculation:*

Eccentricity for Motor Rotor:

Motor Speed, w=16.45 Hz

=987 rpm

$$=\frac{987 \times 2 \times \pi()}{60}$$

=103.59 rad/sec

Balance quality grade=G6.3 as per ISO: 1940-1: 2003 [16]

The dynamic forces are estimated as per the machine balanced grade.

The machined balanced grade is 6.3. The forces shall be estimated by assuming one grade less than that of the relevant machine grade. The next lower grade is G16 So e ω=16 mm/sec

Eccentricity for motor rotor, e=0.154 mm

Eccentricity for Mill & Gear box Rotor:

Mill speed, w=0.55 Hz=33 rpm=3.455 rad/sec

Balance quality grade=G16 as per ISO: 1940-1: 2003 [16].

The dynamic forces are estimated as per the machine balanced grade.

The machine balanced grade is 16he forces shall be estimated by assuming one grade less than that of the relevant machine grade.

The next lower grade is G40.

So e ω=40 mm/sec

Eccentricity for Mill & Gear box Rotor, e=11.575 mm

***Table 15.*** *Unbalance Force Table.*

| Rotor | Weight, N | Unbalanced Force, N |
|---|---|---|
| | | mew^2 |
| Motor | 17000 | 2802 |
| Mill | 210000 | 2901 |

Dynamic Loads Input in Ansys:

Motor:

The centrifugal forces at motor rotor level are transferred onto top deck as below.

Centrifugal force at Motor shaft=2802 N

Real and Imaginary components of the centrifugal force for motor and mill are considered in the directions as shown on the figures below.

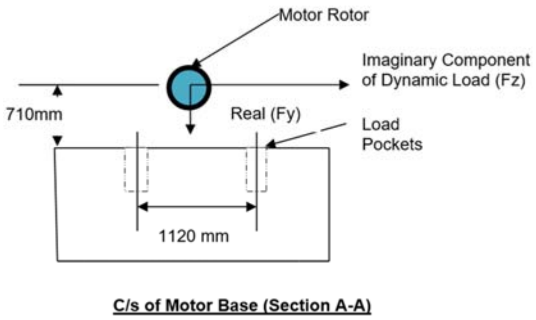


*Figure 11.* *C/s of Motor Base (Section A-A).*

Number of pockets for motor=4

Real component of the centrifugal force on each load pad, Fy=2802/4=701.5 N (Vertical)

Imaginary component of the dynamic load will have horizontal and a couple on the top deck.

Horizontal load due to imaginary component of the centrifugal force on each pocket, Fz=2802/4=701.5 N (Fz)

Couple on top deck, Mx=2802 x 0.710=1990.84 N-m

Push Pull (Fy) due to couple on load pads, Fy=(1990.84/1.12)/2=895.64 N per pocket

Mill:

The centrifugal forces at mill and gear box rotor level are transferred onto top deck as below.

Centrifugal force at mill and gear box table=2901 N

Number of pockets for mill=6

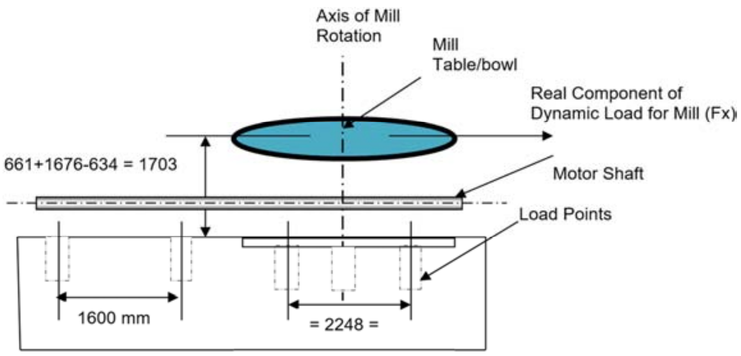


*Figure 12.* *Section B-B.*

Real component of the centrifugal force on each load pad, Fx=2901/6=483.5 N (Fx)

Real component of the dynamic load will have horizontal and a couple of the top deck.

Couple on top deck about Z-axis, Mz=2901 x 1.703=4940.40 N-m

Push Pull (Fy) due to couple on load pads,

Fy=(4940.40/2.248)/2=1098.75 N per pocket

Horizontal load due to imaginary component of the centrifugal force on each pocket, Fz=2901/6=483.5N (Fz)

Couple on top deck, Mx=2901 x 1.703=4904.40 N-m

Push Pull (Fy) due to couple on load pads, Fy=(4904.40/2.248)/3=732.56 N per pocket

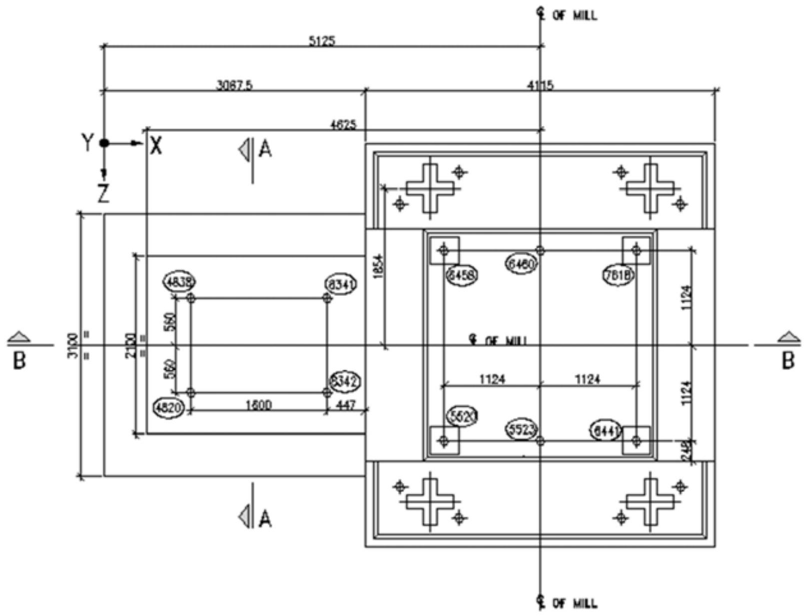


*Figure 13.* *Plan View Of Mill Foundation.*

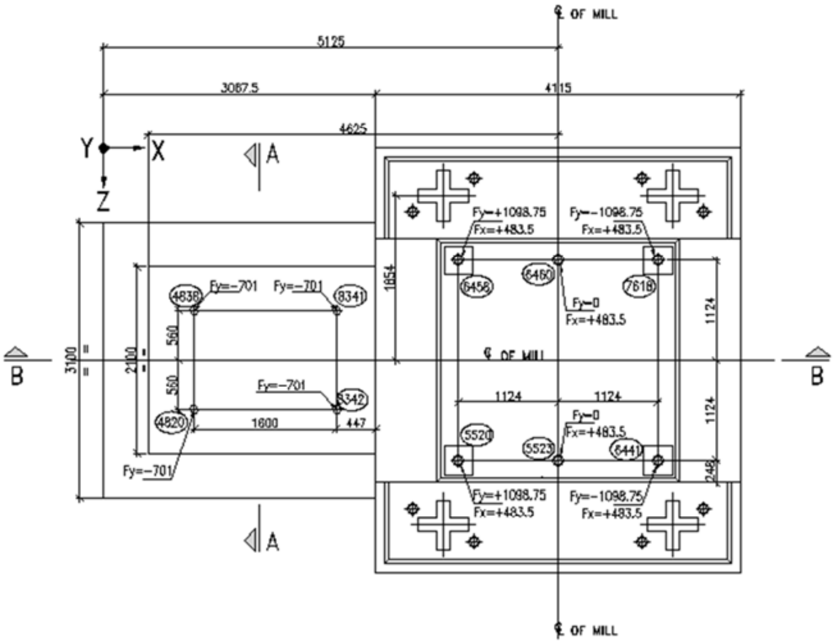


*Figure 14. Dynamic Forces – Real Part (Motor Dynamic Load vertically downward (-Y dir) and Mill Dynamic Load in +X direction).*

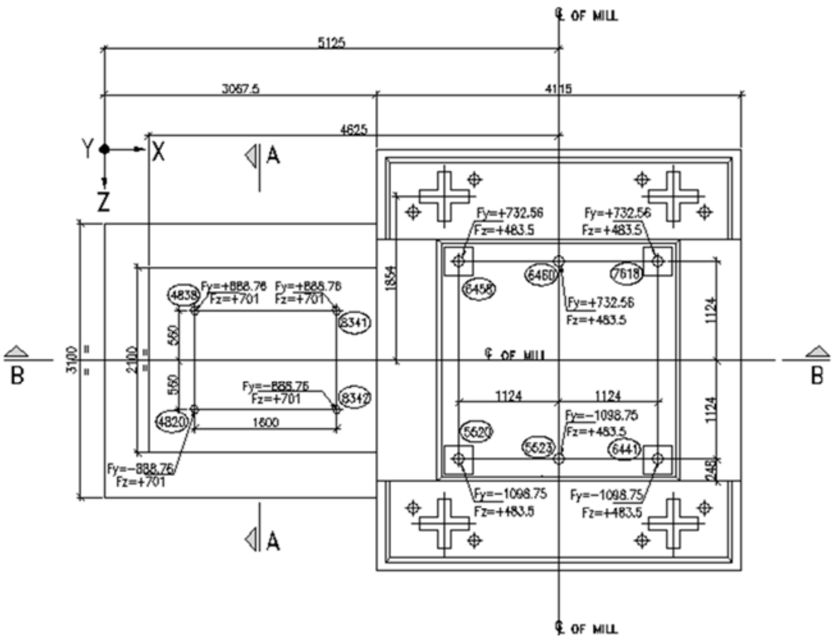


*Figure 15. Dynamic Forces – Imaginary Part (Motor Dynamic Load horizontal (+Zdir) and Mill Dynamic Load in +Z direction).*

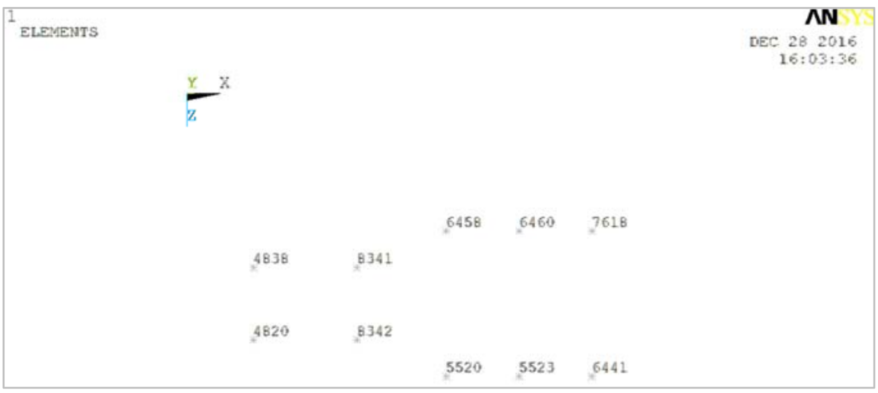


*Figure 16. Node Numbers at Load Points.*

The above dynamic loads act on the rotor at different phase angles at motor and mill locations. However, as a conservative approach, they are assumed to act simultaneously in the same phase at both locations in the same direction and applied accordingly. The displacement amplitudes are obtained for the above worst condition and are compared with the allowable limits.

Referring to displacement amplitude table, it may be noted that the amplitudes are within the allowable limits.

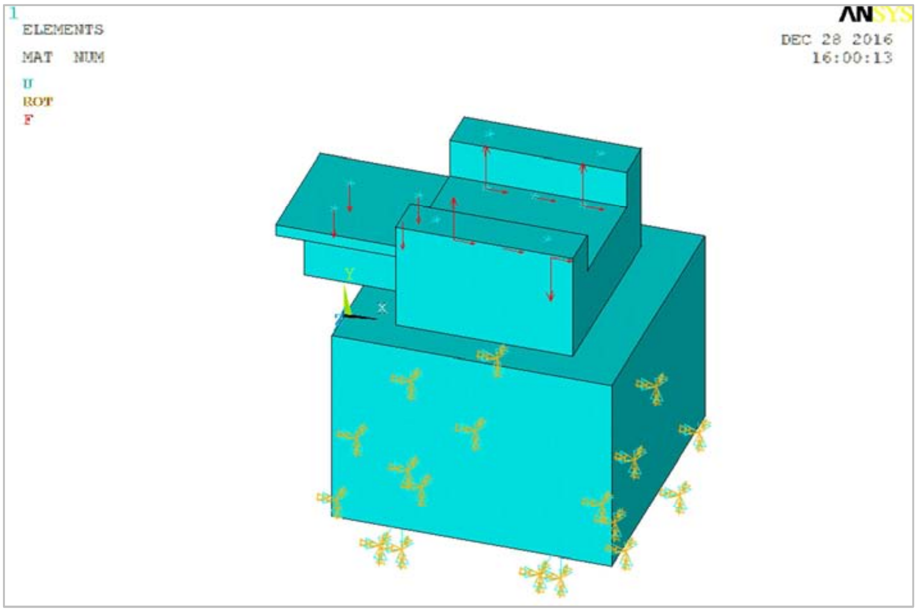


***Figure 17.*** *Applied Dynamic Forces.*

***Table 16.*** *Displacement amplitudes for Operating Frequencies.*

| | Node | Frequency | Amplitude | | |
|---|---|---|---|---|---|
| | | | **Ux, m** | **Uy, m** | **Uz, m** |
| Motor Support Point | 4838 | 0.5 Hz | 1.07E-07 | 5.03E-08 | 3.65E-07 |
| | | 16.5 Hz | 2.13E-07 | 3.86E-07 | 1.72E-06 |
| | 8341 | 0.5 Hz | 1.07E-07 | 3.46E-08 | 3.42E-07 |
| | | 16.5 Hz | 2.14E-07 | 3.33E-07 | 1.53E-06 |
| | 4820 | 0.5 Hz | 1.07E-07 | 4.81E-08 | 3.65E-07 |
| | | 16.5 Hz | 2.03E-07 | 4.63E-07 | 1.72E-06 |
| | 8342 | 0.5 Hz | 1.07E-07 | 3.27E-08 | 3.42E-07 |
| | | 16.5 Hz | 2.05E-07 | 3.92E-07 | 1.53E-06 |
| Mill Support Point | 6458 | 0.5 Hz | 1.10E-07 | 4.48E-08 | 3.29E-07 |
| | | 16.5 Hz | 2.44E-07 | 5.63E-07 | 1.44E-06 |
| | 6460 | 0.5 Hz | 1.10E-07 | 4.28E-08 | 3.13E-07 |
| | | 16.5 Hz | 2.46E-07 | 5.64E-07 | 1.30E-06 |
| | 7618 | 0.5 Hz | 1.10E-07 | 4.45E-08 | 2.97E-07 |
| | | 16.5 Hz | 2.47E-07 | 5.71E-07 | 1.16E-06 |
| | 5520 | 0.5 Hz | 1.10E-07 | 4.36E-08 | 3.29E-07 |
| | | 16.5 Hz | 2.26E-07 | 6.04E-07 | 1.44E-06 |
| | 5523 | 0.5 Hz | 1.10E-07 | 4.31E-08 | 3.13E-07 |
| | | 16.5 Hz | 2.27E-07 | 5.79E-07 | 1.30E-06 |
| | 6441 | 0.5 Hz | 1.10E-07 | 4.62E-08 | 2.97E-07 |
| | | 16.5 Hz | 2.28E-07 | 5.59E-07 | 1.16E-06 |

The limiting vibration amplitude=200μm
(Refer: Clause 5.4.1 of IS: 2974 (Part IV) – 1979. [15]
At motor and mill operating frequencies the amplitudes are < 200 μm

### *6.5. ANSYS – Amplitude vs Frequency Plots*

The following plots show the variance of amplitude (Ux, UY & UZ) with frequency (0~30 Hz) for different nodes at base locations. X & Z are TRANSVERSE and Y is VERTICAL directions of Mill foundation.

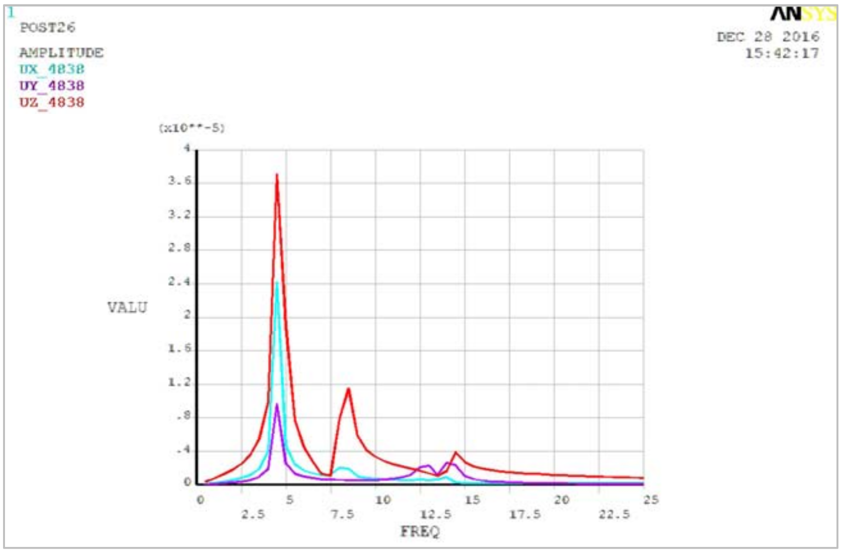


***Figure 18.*** *Vibration Amplitude Plot– Node: 4838_Ux, Uy and Uz.*

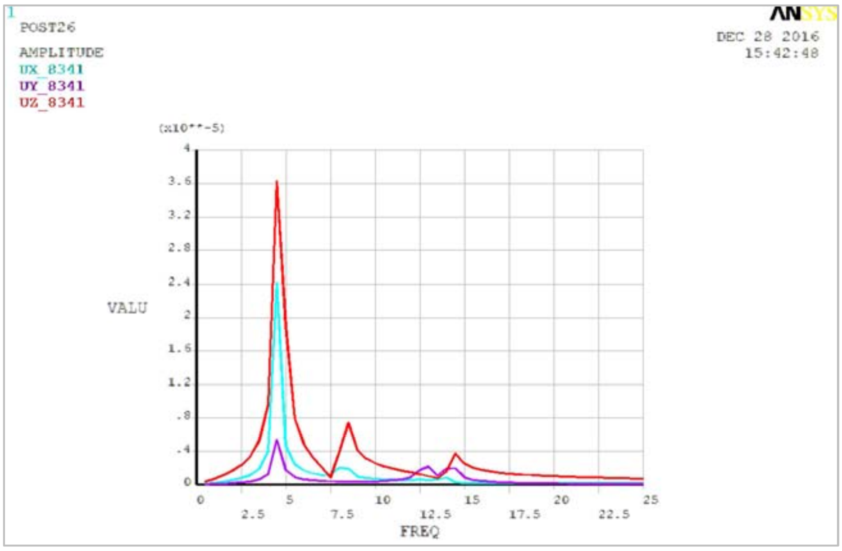


***Figure 19.*** *Vibration Amplitude Plot – Node: 8341_Ux, Uy and Uz.*

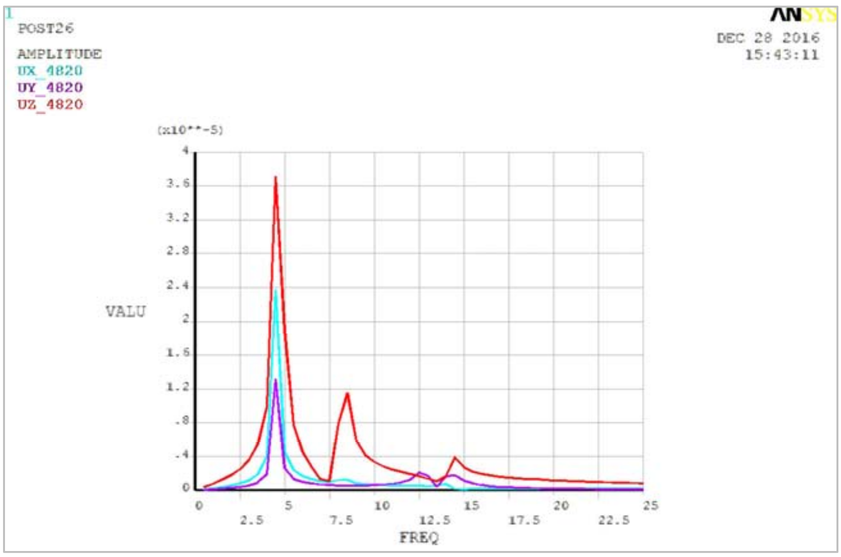


***Figure 20.*** *Vibration Amplitude Plot – Node: 4820_Ux, Uy and Uz.*

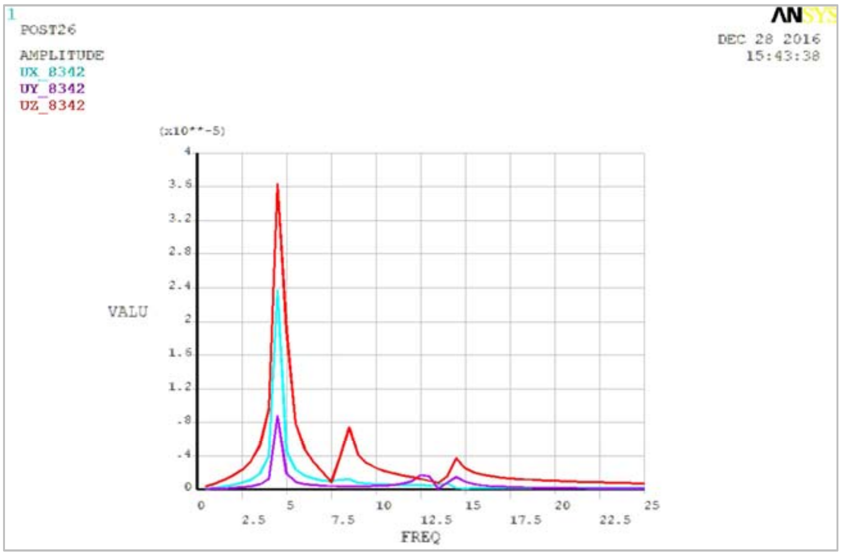


***Figure 21.*** *Vibration Amplitude Plot – Node: 8342_Ux, Uy and Uz.*

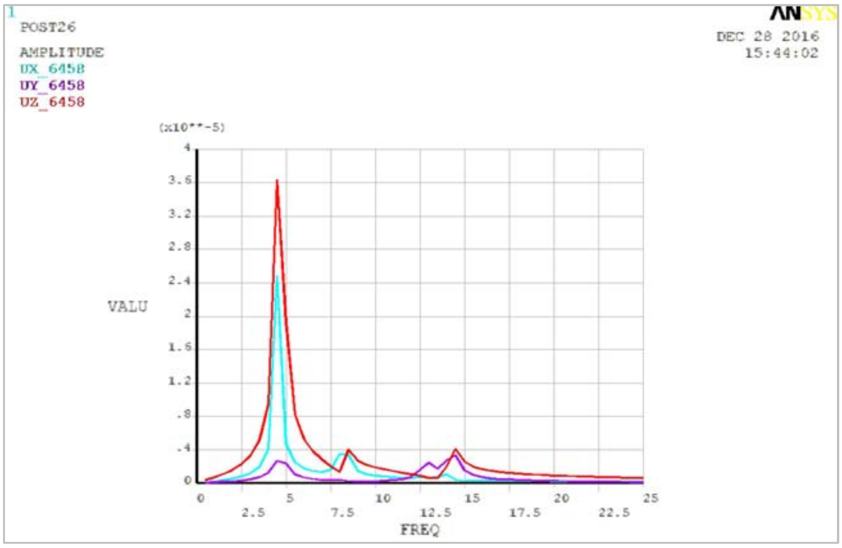


***Figure 22.*** *Vibration Amplitude Plot – Node: 6458_Ux, Uy and Uz.*

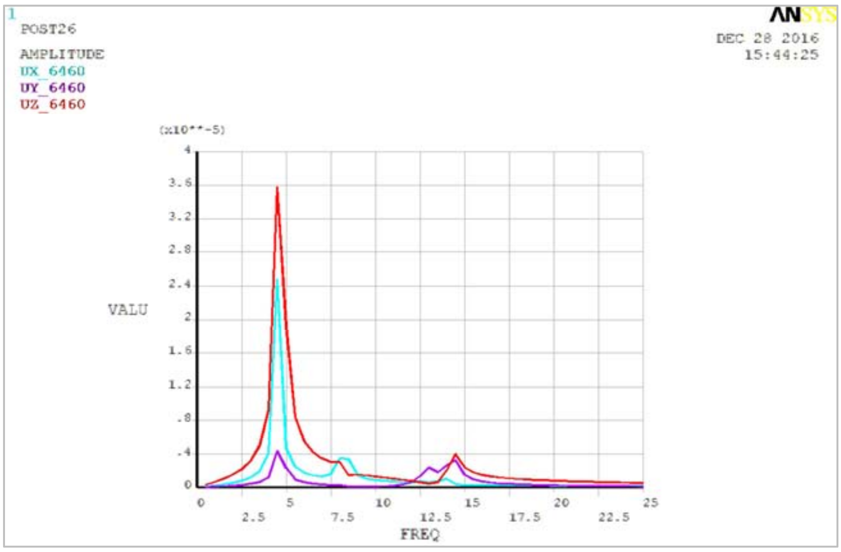


***Figure 23.*** *Vibration Amplitude Plot – Node: 6460_Ux, Uy and Uz.*

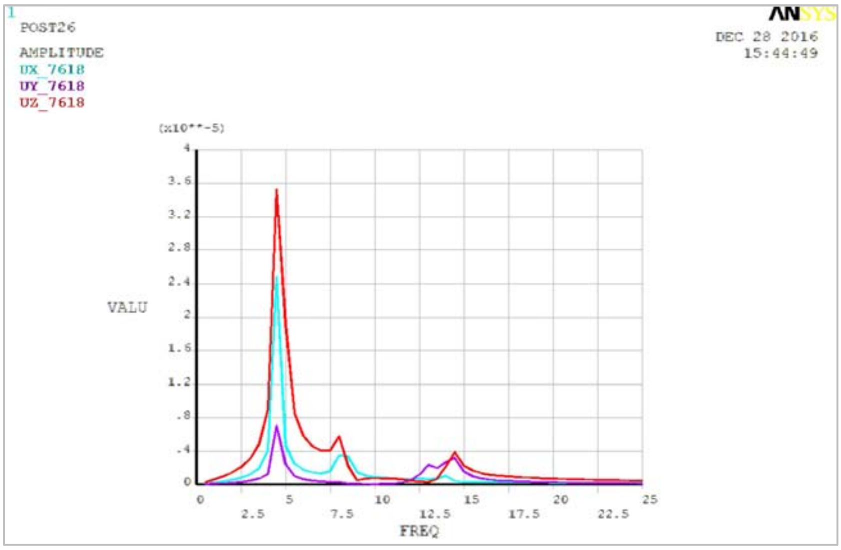


***Figure 24.*** *Vibration Amplitude Plot – Node: 7618_Ux, Uy and Uz.*

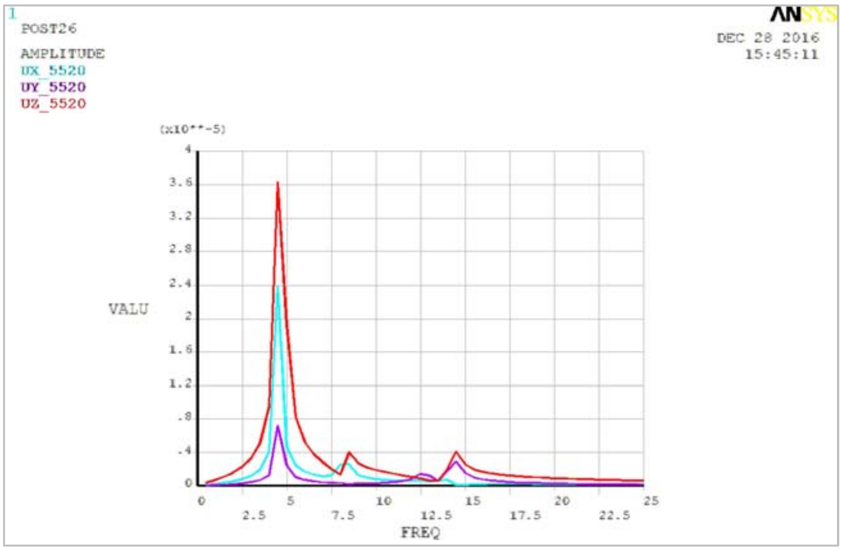


***Figure 25.*** *Vibration Amplitude Plot – Node: 5520_Ux, Uy and Uz.*

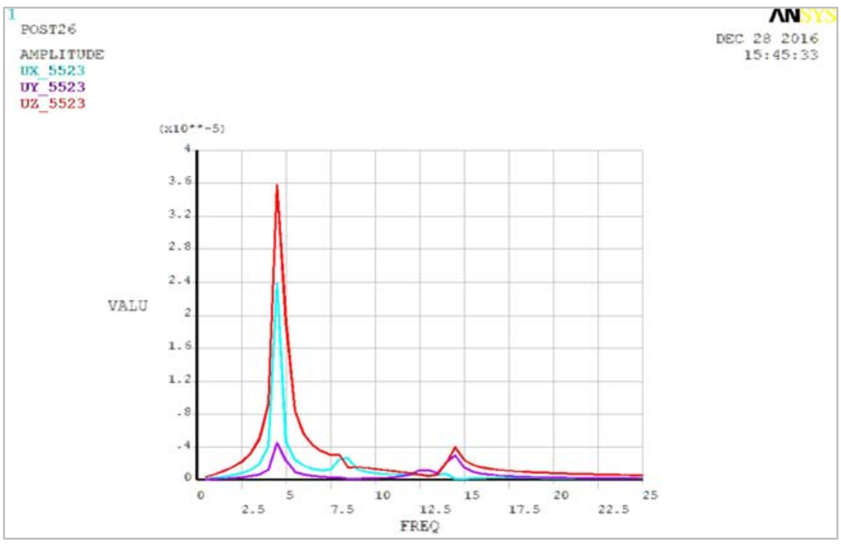


***Figure 26.*** *Vibration Amplitude Plot – Node: 5523_Ux, Uy and Uz.*

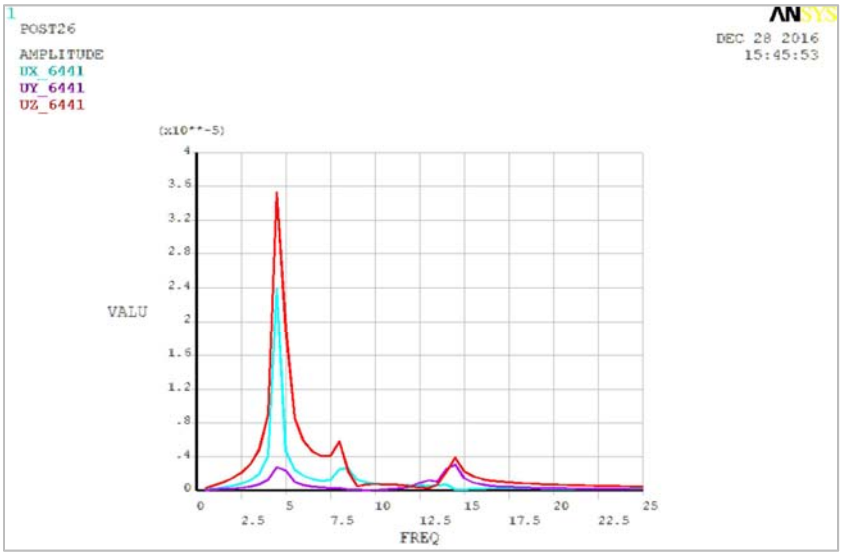


***Figure 27.*** *Vibration Amplitude Plot – Node: 6441_Ux, Uy and Uz.*

## 7. Conclusions

FEM analysis of this type of critical foundations will help for safe & optimum design under static and dynamic loading conditions. These models can also determine the effects of various material characteristics on the stresses and deflections in all directions.

Operating frequency of the mill is 0.55 Hz, The natural frequencies are far away from the Mill frequency range: 0.44Hz (0.8*0.55) To 0.66Hz (1.2*0.55) and Operating frequency of the motor is 16.47 Hz, Motor frequency range: 13.176Hz (0.8*16.47) To 19.764Hz (1.2*16.47) Also, it may be noted that, the motor rotor weight is 17KN which is only $364^{th}$ part of the foundation weight (6194.51KN). In general, if the rotating part weight is one hundredth part or less of its supporting foundation weight, then its dynamic effect on foundation are negligible. Hence motor dynamic effects can be ignored. M35 grade frequencies (i.e. 13.0673 Hz) are very close to 0.8 times of motor frequency (i.e. 0.8 x 16.47=13.176Hz). Therefore, M25 grade of concrete is used for foundation. Frequency vs Amplitude plots for the variance of amplitude (Ux, Uy & Uz) with frequency (0~30 Hz) for different nodes at equipment base locations are generated. These plots helped in understanding the foundation behavior during equipment start-up & shut down transient conditions apart from the operating condition. The peak values shown in the plots represents the transient stage resonant condition's which are many times more than the amplitude values at operating conditions. This helps engineers to consider any precautionary measures during transient conditions. All the amplitudes at base points of motor and mill on the top of the foundation under operating frequencies (16.47 Hz) are well within the allowable value of 200 µm.

The displacements amplitudes arrived from the analysis are very close to realistic behavior of the foundation which emphasizes the importance of application of finite element methods for dynamic analysis of this type of critical machine foundations. This paper will also help the structural engineering practitioners in design of Coal Mill foundation's by applying Finite Element Methods using ANSYS software.

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[1] Han, Y. C. 2008. Study of Vibrating Foundation Considering Soil-Pile-Structure Interaction for Practical Applications. J. of Earthquake Engineering and Engineering Vibration, Vol. 7, No. 3, 321-327.

[2] R. Thompson, D. Wong, Boiler induced draft fan optimization, International Sugar Journal 113 (1350) (2011) 425-431.

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